Sem-IV
PSMT401, PSMT402, PSMT403 and PSMT404
Sample Questions

1. (1 point) The extension $\mathbb{Q}(\sqrt{2})$ over $\mathbb{Q}$ is:
    1. not algebraic
    2. not finite
    3. algebraic
    4. of degree four
2. (1 point) Let $\omega \neq 1$ be a cube root of unity. Then, the degree of the extension $\mathbb{Q}(\omega)$ over $\mathbb{Q}$ is:
    1. Four
    2. Two
    3. Three
    4. One
3. (1 point) The degree of the splitting field of $X^5 - 2$ over $\mathbb{Q}$ is:
    1. Twenty
    2. Ten
    3. Five
    4. One
4. (1 point) Consider the extension $L = \mathbb{Q}(\sqrt[3]{2})$. Then the extension $L/\mathbb{Q}$ is
    1. Not algebraic
    2. Not normal
    3. Not finite
    4. Not separable.
5. (1 point) The degree of the extension $\mathbb{Q}(\sqrt[3]{17}, \sqrt{19})$ over $\mathbb{Q}$ is
    1. Three
    2. Two
    3. Six
    4. One
6. (1 point) Which of these constructions is possible using ruler and compass?
    1. Trisecting angles
    2. Doubling cubes

3. Equilateral triangles
4. Squaring circles

7. (1 point) Let F be a field of characteristic zero. Which of the following statements is true?

    1. Every irreducible polynomial over F is separable.
    2. Every polynomial over F is separable.
    3. Every polynomial and its derivative over F always have a common root.
    4. Every polynomial over F always has a root in F.

8. (1 point) Let $\mathbb{F}_p$ denote the finite field with p elements. Then, the map $\phi : \mathbb{F}_p \to \mathbb{F}_p$ given by $\phi(x) = x^p$ is

    1. Only injective but not surjective
    2. Only surjective but not injective
    3. A bijection
    4. Neither surjective nor injective

9. (1 point) Let $L = \mathbb{Q}(\sqrt{2}, \sqrt{3})$. A primitive element for this extension is:

    1. $\sqrt{2}$
    2. $\sqrt{3}$
    3. $\sqrt{2} + \sqrt{3}$
    4. $\sqrt{6}$

10. (1 point) If the minimal polynomial of $\sqrt{1 + \sqrt{3}}$ is written in the form $X^4 + bX^3 + cX^2 + dX + e$, then the values of c and e are

    1. $c = -2, e = 2.$
    2. $c = 2, e = -2.$
    3. $c = 2, e = 2.$
    4. $c = -2, e = -2.$

11. (2 points) The Fourier series expansion of $f(x) = \tan\theta$ where $\theta \in [0, 2\pi]$ is given by

    1. $f(x) = \tan\theta = \sum_{n=0}^{\infty}(a_n \cos n\theta + b_n \sin n\theta)$ where a_n and b_n denotes the Fourier coefficients,
    2. $f(x) = \tan\theta = \sum_{n=0}^{\infty}(a_n \cos n\theta)$ where a_n denote the cosine Fourier coefficients,
    3. $f(x) = \tan\theta = \sum_{n=0}^{\infty}(b_n \sin n\theta)$ where b_n denote the sine Fourier coefficients,

4. $f(x) = \tan\theta$ cannot express in terms of Fourier series in $\theta \in [0, 2\pi]$.

12. (2 points) Let $\hat{f}(n)$ denotes Fourier coefficient of f. Which of the following inequality/ equality holds for L^2 periodic and integrable function f

    1. $|\hat{f}(n)| \leq \|f\|_1 \leq \|f\|$ for all $n \in Z$,
    2. $|\hat{f}(n)| = \|f\|_1 = \|f\|$ for all $n \in Z$,
    3. $|\hat{f}(n)| = \|f\|_1 \leq \|f\|$ for all $n \in Z$,
    4. $|\hat{f}(n)| \leq \|f\|_1 = \|f\|$ for all $n \in Z$.

13. (2 points) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous periodic function such that

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx = 0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$$

for all $n \in Z^+$ then

    1. f is identically zero,
    2. f is a even function,
    3. f is a odd function,
    4. f is neither even nor odd function.

14. (2 points) Let $S_N(f)$ denotes the N-th partial sum of Fourier series of f and $\{e_n\}$ be an orthonormal set. Let f is integrable function defined on the circle. Which of the following inequality holds for any complex number c_n?

    1. $\|f + S_N(f)\| \leq \|f + \sum_{|n|\leq N} c_n e_n\|$,
    2. $\|f - S_N(f)\| \leq \|f - \sum_{|n|\leq N} c_n e_n\|$,
    3. $\|f - S_N(f)\| \leq \|f + \sum_{|n|\leq N} c_n e_n\|$,
    4. $\|f + S_N(f)\| \leq \|f - \sum_{|n|\leq N} c_n e_n\|$.

15. (2 points) Which of the following property holds for the N-th Dirichlet kernel $D_N(\theta)$?

    1. $\int_{-\pi}^{\pi} |D_N(\theta)| d\theta = 2\pi$,
    2. $\int_{-\pi}^{\pi} |D_N(\theta)| d\theta \geq c\log N$ as $N \longrightarrow \infty$ and c is a constant,
    3. $\int_{-\pi}^{\pi} |D_N(\theta)| d\theta \leq c\log N$ where c is a constant,

4. $\int_{-\pi}^{\pi} |D_N(\theta)| d\theta = c \log N$ where c is a constant.

16. (2 points) The Nth Fejer's kernel $F_N(x)$ is given by

    1. $F_N(x) = \dfrac{\cos^2(Nx/2)}{N \cos^2(x/2)}$,
    2. $F_N(x) = \dfrac{\sin^2(Nx/2)}{N \sin^2(x/2)}$,
    3. $F_N(x) = \dfrac{\sin^2(Nx/2)}{N \cos^2(x/2)}$,
    4. $F_N(x) = \dfrac{\cos^2(Nx/2)}{N \sin^2(x/2)}$.

17. (2 points) Which of the following is not a good kernel?

    1. Dirichlet's Kernel,
    2. Fejer's Kernel,
    3. Poisson Kernel,
    4. Heat Kernel .

18. (2 points) The Poisson kernel $P_r(\theta)$ is given by

    1. $P_r(\theta) = \dfrac{1 + r^2}{1 + 2r \cos\theta + r^2}$,
    2. $P_r(\theta) = \dfrac{1 - r^2}{1 - 2r \sin\theta + r^2}$,
    3. $P_r(\theta) = \dfrac{1 + r^2}{1 + 2r \sin\theta + r^2}$,
    4. $P_r(\theta) = \dfrac{1 - r^2}{1 - 2r \cos\theta + r^2}$.

19. (2 points) Let $u(r, \theta) = (f * \mathrm{Pr})(\theta)$, where f is an integrable function defined on the unit circle and $\mathrm{Pr}(\theta)$ denotes the Poisson kernel. If θ is any point of continuity of f then

    1. $\lim_{r \to 1} u(r, \theta) = f(\theta)$,
    2. $\lim_{r \to 1} u(r, \theta) = f(0)$,
    3. $\lim_{r \to 1} u(r, \theta) = 0$,
    4. $\lim_{r \to 1} u(r, \theta) \longrightarrow \infty$.

20. (2 points) The solution of Dirichlet problem $\triangle\, u = 0$ for the unit disc defined by $D = \{(r, \theta)/0 \leq r < 1, 0 \leq \theta < 2\pi\}$ subject to the fixed temperature $\sin\theta$ along the circumference $C = \{(r, \theta)/r = 1, 0 \leq \theta < 2\pi\}$ is given by

1. $\cos\theta$,
2. $r\cos\theta$,
3. $\sin\theta$,
4. $r\sin\theta$.

21. (3 points) Let A be a $n \times n$ matrix. Determinant is
    1. n^2 tensor
    2. n tensor
    3. $2n$ tensor
    4. not a tensor

22. (3 points) Which of the following statement is true:
    1. $T \otimes S = -S \otimes T$
    2. $-(T \otimes S) = (-S) \otimes T$
    3. $-(T \otimes S) = S \otimes (-T)$
    4. $T \otimes S \neq S \otimes T$

23. (3 points) If $v = (2, -1, 0)$ and $w = (0, 1, 3)$ then cross product $v \times w$ is
    1. (0, -1, 0)
    2. (3, 6, -2)
    3. (-1, -2, 1)
    4. (-3, -6,2)

24. (3 points) If $f(x, y, z) = x^2y - xz$ then gradient of f i.e., ∇f is
    1. $(2xy, -z, 0)$
    2. $(2xy, x^2, -x)$
    3. $(-z, x^2, -1)$
    4. $(2xy - z, x^2, -x)$

25. (3 points) If $\omega \in \Lambda^k(\mathbb{R}^n)$ and $\eta \in \Lambda^r(\mathbb{R}^n)$ then $\omega \wedge \eta$ is in
    1. $\Lambda^k(\mathbb{R}^n)$
    2. $\Lambda^r(\mathbb{R}^n)$
    3. $\Lambda^{kr}(\mathbb{R}^n)$
    4. $\Lambda^{k+r}(\mathbb{R}^n)$

26. (3 points) If $f(x, y, z) = xy + z^2$ then df is
    1. 0

2. $(x + y + 2z)dxdydz$
3. $xdx + ydy + 2zdz$
4. $ydx + xdy + 2zdz$

27. (3 points) If $\eta = 3xdy$ and $\omega = xydx + dy$ then $\eta \wedge \omega$ is
    1. $3x\,dy$
    2. $3x^2y\,dx \wedge dy$
    3. $3xy\,dx \wedge dy$
    4. $-3x^2y\,dx \wedge dy$

28. (3 points) Let $F : \mathbb{R}^3 \to \mathbb{R}^3$ and $G : \mathbb{R}^3 \to \mathbb{R}^3$ be vector fields defined by $F(x, y, z) = (x, 0, z)$ and $G(x, y, z) = (y, 0, 0)$. Then $F \times G(x, y, z) =$
    1. (0, 0, 0)
    2. (0, -yz, 0)
    3. (xy, 0, 0)
    4. (0, yz, 0)

29. (3 points) If $\omega = 2zdx + 3ydy$ then $d\omega =$
    1. $2zdx \wedge dz$
    2. $2dx \wedge dz + 3dy$
    3. $2dx + 3dy$
    4. $2dz \wedge dx$

30. (3 points) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = x + y$. The pull back $f^*\omega$ of the form $\omega = dx$ is
    1. $dxdy$
    2. dx
    3. $dx \wedge dy$
    4. $dx + dy$

31. (4 points) Which of the following points is contained within the feasible region corresponding to the system of inequalities? $3x + 6y < 12$
    $5x + 7y > 8$
    1. $(2, 1)$
    2. $(2, 0)$
    3. $(1, 0)$
    4. $(0, 1)$

32. (4 points) Consider the following maximization problem and select the correct number of slack variables required to solve the problem using the simplex mmethod

$$Maximize \quad P = 4 + 4y - 2z$$

subject to

$$x + 2y - 3z \leq 4$$
$$5x + 6y + 7z \leq 8$$
$$9x + 10y + 11z \leq 12$$
$$13x + 14y + 15zz \leq 16$$
$$x \geq 0, y \geq 0, z \geq 0$$

1. 4 slack variables
2. 3 slack variables
3. 1 slack variables
4. 7 slack variables

33. (4 points) At which point is the function $F = 3x + 4y$ minimized with respect to the feasible region

$$-3x + y \leq 0$$
$$2x + y \geq 10$$
$$x \geq 0, y \geq 0$$

1. $(5, 0)$
2. $(0, 0)$
3. $(2, 6)$
4. $(0, 10)$

34. (4 points) After completing the simple method, the solution can be read from the final simplex table. Which variables are not automatically set equal to zero?

1. Non -Basic variables
2. Basic variables
3. Independent variables
4. Dependent variables

35. (4 points) The objective function and constraints are functions of two types of variables, —— variables and ———variables.

1. Controllable and uncontrollable
2. Positive and negative

3. Strong and weak
4. None of the above

36. (4 points) Operations management can be defined as the application of ——————to a problem within a system to yield the optimal solution
    1. Suitable manpower
    2. Financial operations
    3. mathematical techniques, models, and tools
    4. diagrammatic models

37. (4 points) When applying the Golden Section Search method to a function $f(x)$ to find its maximum, the $f(x) > f(x)$ condition holds true for the intermediate points x_1 and x_2 Which of the following statements is incorrect?
    1. The new search region is determined by $[x_1, x_2]$
    2. The upper bound x_u stays the same
    3. The Intermediate point x_1 stays as one of the intermediate points
    4. The new search region is determined by $[x_1, x_2]$

38. (4 points) Using the Golden Section Search method, find two numbers whose sum is 90 and their product is as large as possible. Conduct two iterations on the interval $[0, 90]$.
    1. 30 and 60
    2. 38 and 52
    3. 45 and 45
    4. 20 and 70

39. (4 points) Which of the terms is not used in a linear programming problem
    1. Slack variables
    2. Concave region
    3. Objective function
    4. Feasible solution

40. (4 points) For what value of x , is the function $f(x) = x^2 - 2x - 6$minimized?
    1. 0
    2. 5
    3. 1
    4. 3