UNIVERSITY OF MUMBAI

Syllabus

for M.Phil / Ph. D Course work

(CBCS)

Program: M.Phil / Ph. D

Course: Mathematics

with effect from the academic year 2020-2021

Eligibility for admission to the M. Phil / Ph. D (Mathematics) :-

The candidates who have passed the PET (Mathematics) conducted by University of Mumbai or have passed the NET/SET examination or have a M. Phil. (Mathematics) degree as per the UGC guidelines are eligible to appear for an Interview for the M. Phil / Ph. D.(Mathematics) Programme. The candidates who have qualified in the Interview are be eligible for admission to the M. Phil / Ph. D (Mathematics) Programme.

M. Phil / Ph. D Course work :-

The course work for the M. Phil / Ph. D. has been designed by the latest guidelines by the UGC for minimum standard for the award of M. Phil / Ph. D degree regulations 2016 and as per VCD 2018.

1. Each candidate after having been admitted to the M. Phil / Ph. D. degree programme shall be required to undertake course work for a minimum period of one semester and maximum as per VCD 2018.. The course work shall be treated as pre- Ph. D. preparation.

2. The Ph. D. course work shall be offered with Choice Based Credit System. The candidate will have to earn 12 credits.

3. The candidate is allowed to take the advantage of Credit Transfer from other recognized university or the university department. In consultation of Board of Studies, the Head / Director is authorized to allow the candidate to avail this facility on a case by case basis.

4. Seven point scale will be followed for assigning the final grade. Candidate should get minimum ‘B+’grade to qualify.

5. The Ph. D candidate admitted in the other centers are also eligible to admit for this course work with a prior consent from the Head of the University Department and written permission of the HOD/ Director/Principal of the center.

6. After completion of the course work Head/ Director will issue the certificate of completion in following format.

Department of Mathematics, Mumbai University
This is to certify that Mr/Ms/Mrs.....(Surname) .....(First name).....… (Second name)..... has been a regular student of M. Phil / Ph.D. with registration number .....He/She attended the course work conducted at the recognized research centre /department from .....to.....… during the year ..... He / She has successfully completed the M. Phil / Ph. D course work prescribed by the University of Mumbai. He / She secured … grade in seven point scale.
Date:- Head of the Department / principal
Seal

Course Structure
Work load / Course: 60 Hours = 30 Theory + 30 Tutorials /Practicals

Title of Course	Course Code	Maximum Credits	Mode of Evaluation
Compulsory course-I Research Methodology	MPHIL/PHD 101	4	1. Mid semester test= 40 marks 2. One seminar =10 marks 3. Final test= 50 marks
Compulsory course -II Algebra, Analysis, Topology , Discrete Mathematics	MPHIL/PHD 102	4	As above
Compulsory course -III Guide Course (Advanced Courses)	MPHIL/PHD 103	4	As above
Total		12	

Programme Specific Outcome:

1. Students will learn about LATEX, Mathematical software and use of mathematical softwares in their research work.
2. Students will be able to understand about technical communication and plagiarism
3. Students will be able to frame and solve advanced mathematical problems.
4. Students will be able to demonstrate mastery of subject material, as evidenced by quality of performance in coursework, and on written and oral examinations in mathematics.
5. Students will be able to communicate mathematical ideas, results, context, and background effectively and professionally in written and oral form.
6. Students will be able to produce and defend an original contribution to knowledge, as evidenced by the writing and defence of a dissertation / thesis involving significant original research as per UGC guidelines.

Compulsory Course-I (4 credits, 60 Hours = 30 contact hours + 30 Tutorial /Practicals)

Compulsory Course–I, Course code: MPHIL/PHDM 101 (Research Methodology) (4 Credits)

The said Paper consists of:
1. Latex and Beamer
2. At least one Mathematics software Maxima / Scilab / MATLAB / any other such software offered by a research guide.
3. Technical communication and plagiarism.

Compulsory Course-I (4 credits, 60 Hours = 30 contact hours + 30 Tutorial /Practicals)

Compulsory Course-II (4 credits, 60 Hours = 30 Theory + 30 Tutorials)

Course code : MPHIL/PHD 102 (4 Credits)

Any one of the following Courses:
1. Algebra
2. Analysis
3. Topology
4. Discrete Mathematics
5. Any one semester course designed and offered by a research guide / external expert.

Compulsory Course-I (4 credits, 60 Hours = 30 contact hours + 30 Tutorial /Practicals)

Compulsory Course-III (4 credits, 60 Hours = 30 Theory + 30 Tutorials)
Course code : MPHIL/PHD 103 (4 Credits)

This Paper consists of any of the following equivalents:
1. ATM School participation
2. Review of a minimum of two research papers under the guidance of a teacher
3. A Reading Course under the guidance of a teacher.

Teaching Pattern:

There are Two Theory lectures and Two Tutorials per week per Course (1 lecture / period is of One hour duration)

Detailed Syllabus for Course code : MPHIL/PHD 102 (4 Credits)

1. Algebra

Ideals, Local rings, Localization of rings and modules, Applications. Noetherian modules, Primary decomposition, Artinian modules, Length of a module.
Integral element, Integral extension, integrally closed domain, Finiteness of integral closure.
Valuation rings, Discrete valuation rings, Dedekind domains.

References
[1]. N. Jacobson, Basic Algebra, Vol I & II, Hindustan Publishing Corporation, New Delhi.
[2]. D. S. Dummit, R. M. Foote, Abstract Algebra, John Wiley & Sons, Singapore.
[3]. M. F. Atiyah and I. G. Macdonald, Introduction to commutative Algebra, Addison-Wesley,

Reading.
[4]. N. S. Gopalkrishnan, Commutative Algebra, Oxonian Press Pvt. Ltd, New Delhi.
[5]. S. Lang, Algebra, Addison-Wesley Publishing Company, Singapore.

2. Analysis

C(X)-Spaces of continuous functions on a metric space X; the topologies of C(X) with respect to X compact, locally compact (and Hausdorff) cases; Discussion on norms, seminorms, induced topology. Discussion of C_c (X), C_o (X) etc. Discussion of Weierstrass theorem and its different proofs; Stone Weierstrass theorem, Tietze's extension theorem.

Normed linear spaces; Discussion of Finite and infinite dimensional cases. Banach spaces, closed graph theorem, open mapping theorem, Uniform boundedness principle, Banach Steinhaus theorem; Equicontinuity. Inner product spaces, Hilbert spaces, orthonormal basis, Direct Sum, Dual space of a Hilbert space. Riesz representation theorem.

Brief treatment of Lebesgue measure and Integral in general setting; Lp spaces, completeness, Duality , Reflexivity. Detailed study of L^1 (0, 2π] and L^1 (R) and its connection to Fourier Analysis. Fourier transform on L^1 ; Poisson Summation formula; Fourier inversion formula, Riemann Lebesgue Lemma, Fourier transform on L^2 ; Parseval's Identity, Plancherel theorem. The Schwartz class S of rapidly decreasing functions and its topology; Tempered distributions T and its topology; Fourier transform as a bijection on S and T.

References
[1]. Richard Beals, Analysis- An Introduction, Cambridge University Press.
[2]. E. Hewitt and K. Stromberg, Real and Abstract Analysis, Springer- Narosa.
[3]. J. B. Conway, A course in Functional Analysis, Springer Graduate Texts in Mathematics.
[4]. Walter Rudin, Functional Analysis, McGraw Hill.
[5]. R. Strichartz, A guide to Distribution Theory and Fourier Transforms, CRC Press.
[6]. Kolmogorov and Fomin, Measure, Lebesgue Integral and Hilbert Space, Academic Press.

3. Topology

Homotopy, Fundamental Group, Homotopy Equivalence.
Covering Spaces, Classification of Covering Spaces, Covering spaces and Fundamental Group, Covering Transformations.
Classification of Surfaces, Seifert-van Kampen Theorem and applications.

References

[1]. James R. Munkres, Topology, Pearson Education, New Delhi.
[2]. John M. Lee, Introduction to topological manifolds, Springer-Verlag, new York.
[3]. W. S. Massey, Algebraic Topology an introduction, Harcourt Bruce& World Inc, New York.
[4]. Allen Hatcher, Algebraic Topology, Cambridge University Press.

4. Discrete Mathematics

Pigeon hole principle, Ramsey Theory, Some bounds, Addition and Multiplication principles, counting Techniques, Elementary graph Theory, Connected, Eulerian, Hamiltonian graphs, Theorems of Dirac and Posa, Ford-Fulkerson Theorem, Matching Algorithm, Hungarian Algorithm, Eigenvalues of graph, Directed graphs. In addition one of the following topics:

1.Advance graph theory
2.Algorithmic graph theory
3.Generalize Quadrangles
4.Tournaments

References

[1]. Biggs, Norman, Algebraic Graph Theory, Cambridge University Press.
[2]. Godsil, Chris; Royle, Gordon , Algebraic Graph Theory, GTM 207, Springer-Verlag.
[3]. Douglas B. West, Introduction to Graph Theory, Prentice Hall India.
[4]. William Kocay, Donald L. Kreher, Graphs, Algorithms and Optimization, Chapman and Hall.

Detailed Syllabus for Course code : MPHIL/PHD 103 (4 Credits)

1. Commutative Algebra

Modules, Free modules, Exact sequences, Projective modules, Injective modules, Tensor products, Flat modules.
Filtered rings and modules, Completion, I- adic filtration, Associated graded rings.
Complexes, Derived functors, Homological dimension.

Reference Books

[1]. N. Jacobson, Basic Algebra, Vol I & II, Hindustan Publishing Corporation, New Delhi.
[2]. D. S. Dummit, R. M. Foote, Abstract Algebra, John Wiley & Sons, Singapore.
[3]. M. F. Atiyah and I. G. Macdonald, Introduction to commutative Algebra, Addison-Wesley,

Reading.

[4]. N. S. Gopalkrishnan, Commutative Algebra, Oxonian Press Pvt. Ltd, New Delhi.
[5]. S. Lang, Algebra, Addison-Wesley Publishing Company, Singapore.

2. Advanced Analysis

Detailed study of one of the following topics:

1. Operator theory on Hilbert spaces and Spectral analysis.
2. Classical Differentiation, Dini's derivatives, Functions of Bounded Variation, Absolute conti nuity, Decomposition of measures, Radon Nikodym derivative theorem and Radon Nikodym theorem; Its applications to Financial Mathematics.
3. Hilbert space techniques, orthonormal basis, theory of wavelets, multiresolution analysis, wavelet basis for L^2 .
4. Theory of Distributions, elliptic PDE.
5. Harmonic Analysis on locally compact abelian groups.
6. Brownian motion, Ito integral, Stochastic Differential equations and Application to Financial Mathematics.
7. Several Complex Variables.
8. Ergodic theory

Reference Books

[1]. Richard Beals, Analysis- An Introduction, Cambridge University Press.
[2]. E. Hewitt and K. Stromberg, Real and Abstract Analysis, Springer- Narosa.
[3]. J. B. Conway, A course in Functional Analysis, Springer Graduate Texts in Mathematics.
[4]. Walter Rudin, Functional Analysis, McGraw Hill.
[5]. R. Strichartz, A guide to Distribution Theory and Fourier Transforms, CRC Press.
[6]. Kolmogorov and Fomin, Measure, Lebesgue Integral and Hilbert Space, Academic Press.
[7]. Gilbert G. Walter, Wavelets and other Orthogonal Systems with Applications, CRC Press.
[8]. H. I. Resnikoff and R. O. Wells, Wavelet Analysis, the scalable structure of Information, Springer.
[9]. I. Karatzas and S. E. Shreve, Brownian Motion and Stochastic Calculus Applied to Finance, Springer.
[10]. Lamberton and Lapeyre, Introduction to Stochastic Calculus Applied to Finance, Chapman and Hall.

3. Advanced course in Topology

Simplicial Complexes, Triangulable spaces, examples, Abstract simplicial complexes, Simplicial approximation theorem, Simplicial Homology, applications.
Singular homology, Homotopy Invariance.
Homology and the Fundamental Group, Meyer-Vietoris theorem, applications.

Reference Books

[1]. James R. Munkres, Topology,
[2]. John M. Lee, Introduction to topological manifolds,
[3]. W. S. Massey, Algebraic Topology an introduction,
[4]. Allen Hatcher, Algebraic Topology, Cambridge University Press.

4. Advances in Discrete Mathematics

Finite fields, Finite geometries, Projective planes, Affine planes, Difference sets, Designs, Construction of Design, Symmetric Design, Binary linear Codes, Generator matrix and check matrix, decoding, Spheres packing, Gilbert-Varshamov bound, single error correcting

Hamming Codes. In addition one of the following topics:

1.Advance Coding Theory
2.Matroid Theory
3.Game Theory
4.Combinatorial Matrix Theory

Reference Books

[1]. Thomas Beth & D. Jungnickel & H. Lenz, Design Theory, Volume 1 & 2 Encyclopedia of mathematics and its applications.
[2]. W. D. Wallis, Further Computational and Constructive Design Theory, Kluwer Academic Publishers.
[3]. Peter Dembowski, Finite Geometries, Classics in Mathematics, Springer.
[4]. J. H. van Lint, Introduction to Coding Theory, Springer.
[5]. Florence Jessie MacWilliams & Neil James Alexander Sloane, The theory of Error Correcting Codes, North-Holland.
[6]. Richard A. Brualdi, Drago M. Cvetkovi, A Combinatorial Approach to Matrix Theory and its Applications, Chapman & Hall/CRC Press.
[7]. Richard A. Brualdi, Herbert John Ryser, Combinatorial Matrix Theory, Cambridge University Press.
[8]. Guillermo Owen, Game Theory. Academic Press.
[9]. Philip D. Straffin, Game theory and Strategy, Mathematical Association of America Textbook.

5. Fractional Calculus: Theory and Applications

An Introduction to Fractional Calculus: Gamma function, Mittag_Leffler functions, Hypergeometric functions, The fractional Leibnitz product rule, Differintegral, The fractional derivatives in terms of finite differences- the Grunwald-Letnikove derivative, the Grunwald-Letnikove Differ-integral of arbitrary order, Abel's integral equation, Differential representation of the Riemann and Caputo fractional derivative/ integral, Laplace transform of R-L integral/derivative, The Riesz fractional derivative.

Analysis of Fractional Differential Equations: Fractional differential equations involving Caputo derivatives, Adomian decomposition method, Adomian polynomials, Convergence of Adomian series, Application of ADM, New iterative method, Convergence and application of NIM.

Fractional Boundary Value Problem: Fractional initial and boundary value problem, New iterative method for FBVP, Separation of variables method, Existence and uniqueness theorem for initial value problems, Continuous dependence on initial conditions, Continuous dependence on the order of differentiation.

Applications of Fractional Calculus: The fractional harmonic oscillator, The fractional harmonic oscillator according to Fourier, The fractional harmonic oscillator according to Riemann, The fractional harmonic oscillator according to Caputo, Fractional wave equation, Fractional heat equation.

Reference Books

[1] .Fractional Calculus: Theory and Applications by Varsha Daftardar-Gejji, Narosa
[2]. Fractional calculus: an introduction for Physicists, 2nd edition by Richard Herrmann, World Scientific.

6. Algebraic Number Theory:

Number fields: Field extensions, number fields, Algebraic numbers, Integral extensions, Ring of integers in a number field,

Quadratic Reciprocity: The Legendre symbol, Jacobi symbols, Law of quadratic reciprocity.

Quadratic fields and factorization: Quadratic fields, real and imaginary quadratic fields, rings of integers in a quadratic field, the group of units, ideal √ factorization in a quadratic field, examples like Z[i], Z[√−5]. Factorization of rational primes in ring of integers of quadratic number fields.

Class Group: The ideal class group, definition and examples in quadratic number fields, class groups of imaginary quadratic number fields, Minkowski lemma, finiteness of class group, computation of class group, applications to Diophantine equations.

Reference books:

[1] TIFR Pamphlet, Algebraic Number Theory.
[2] Artin, Algebra.
[3] Niven, Zuckermann, Theory of Numbers.

7. Arithmetic of p-adic integers:

Finite fields: all details like existence and uniqueness, split-ting field, construction, factorization. Results like multiplicative group of a finite field is cyclic using structure theorem as well as Euler's Phi function.

Equations over finite fields, Chevalley Warning theorem, Quadratic Reciprocity law.

p-adic fields: the ring Z_p and the field Q_p . Exact sequences, valuation ring, discrete valuation ring, archimedean valuations, nonarchimedean valuations, Ostrowski's theorem,
All triangles in a field with nonarchimedean valuation are isosceles.

Topological properties of Z_p and Q_p . Hensel's lemma, Multiplicative group of Q_p , Hilbert symbol, Hilbert product formula (with proof), Hasse-Minkowski theorem.

Reference books:

[1] Artin, Algebra.
[2] Koblitz N., p-adic numbers, p-adic analysis and zeta functions.
[3] Gouvea F., p-adic numbers, An introduction.

8. Basic introduction to Lie Algebras

Definition and examples of Lie algebras, Lie algebra of derivations, adjoint representation, ideals and homomorphisms, solvable and nilpotent Lie algebras.

Engel's theorem, Lie's theorem and applications.

Cartan's criterion for solvability, Killing form and Criterion for semisimplicity, inner derivations,

semisimple Lie algebras.

Root systems, Bases, Existence of Base, Coxeter graphs and Dnykin diagrams examples.

Reference books:
[1] Humphreys, An introduction to Lie Algebras.
[2] Carter, Lie algebras of finite and affine type.

9. Stability Theory

Autonomous equations: Phase space, orbits, critical points and linearisation, periodic solutions, first integrals and integral manifolds, Liouville's theorem.

Critical Points and Periodic solutions: Two dimensional linear systems, remarks on three dimensional systems, critical points of non-linear equations, Bendixson's criterion, the Poincaré Bendixson's theorem, periodic solutions in R^n.

Theory of Stability: Simple examples, stability of equilibrium solutions, stability of periodic solutions, linearisation, equations with constant coefficients, equations with coefficients which have a limit, equations with periodic coefficients.

Stability by linearisation: Asymptotic stability of the trivial solution, instability of trivial solution, stability of periodic solutions of autonomous equations.

Reference Books:
[1]. Ferdinand Verhulst, Nonlinear Differential Equations and Dynamical Systems, Second edi tion, Springer-Verlag, 1996.
[2]. Lawrence Perko, Differential Equations and Dynamical Systems, Third Edition, Springer-Verlag, 2001.

10. Linear and Nonlinear Waves

Waves, First order hyperbolic equations and its solution by the method of characteristics, kinematic waves, shock waves, shock structure, conservation laws and weak solutions, shock fitting with respect to quadratic $Q(\rho)$, some specific problems like traffic flow problem, flood waves etc.

Burger's equation and its solution by Cole Hopf transformation. Hyperbolic systems, their characterization and classification with examples.

Gas Dynamics: Equations of motion, Thermodynamic Relations, Nonlinear Plane waves, the corresponding characteristic equations, shock waves, useful forms of shock conditions, weak and strong shock conditions.

Reference Books:
[1]. Linear and Nonlinear Waves, G. B. Whitham, Wiley.
[2]. Shock waves and Reaction Diffusion Equations, Joel Smoller, Springer Verlag
(Part III - The Theory of Shock waves).

Note: In addition the research guides may be allowed to run courses specific to their current research interests also such as.

11. Partial Differential Equations
12. Advanced topics in Functional Analysis
13. Analysis on Heisenberg groups

Scheme of Examination

The scheme consists of seminars /assignments and two tests in a semester. Each candidate will have to submit a Dissertation which will be assessed by an external examiner.
Attendance: 75% attendance to the lectures is essential to qualify for appearing for the tests.
First Test: There will be a mid-semester written test for 40 marks on the topics covered of 2 hours duration in all the three courses.

Seminars: There will be continuous assessment in the form of Seminars /Lab works and assignments during the semester and 10 marks are be allotted for this in each course.

Final Test: The final test is for 50 marks and is of 2 hours duration in each course on the entire syllabus of the semester.

Candidates securing a total of 50 or more marks in a course from the first test, seminar/ Lab works and the final test with at least 25 marks in the final test will be declared to have passed the respective course and earned 3 credits. Candidates failed in a course will be allowed to take a supplementary examination (at most two attempts) for the final test in the respective course. If a candidate participates in a ATM work shop as course 3 in semester II, then the Head of the Department will request the Coordinator of the ATM work shop to give a grade to the candidate based on the participation.

A candidate who secures a total of **12 credits** from the three courses will be declared to have passed the M. Phil / Ph. D Course work and he/she will be allowed to submit the Ph. D./M. Phil. dissertation. There is no grade point for the Dissertation work. The Dissertation work is accepted or rejected.

The candidates selected for the Ph. D. programme, after acquiring the M. Phil. (mathematics) degree of University of Mumbai need no credits to be earned. Such candidates will be allowed to submit the Ph. D. dissertation. The candidates selected for the Ph. D. programme after acquiring the M. Phil. (Mathematics) degree of any other University will have to earn 12 credits if there is no equivalent course work prescribed for the M. Phil. programme of the respective other University.

If a candidate wants to leave the Ph. D. programme after earning 12 credits in the Ph. D Course work, he/she may be allowed to register for the M. Phil. Programme. In such a case, the 12 credits will be transferred to his/her M. Phil. Programme and he/she will be allowed to submit the Dissertation.

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